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Stability of a circular cylinder oscillating in uniform flow or in a wake

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The lift and drag forces were measured on both a single circular cylinder and tandem circular cylinders in uniform flow at Reynolds numbers from 40 to 10^4, to investigate the stability of an oscillating cylinder. A cylinder (the downstream one in the tandem case) was made to oscillate in either the transverse or longitudinal direction (perpendicular or parallel to the stream). In the case of a single cylinder, its oscillation causes the so-called synchronization in a frequency range around the Strouhal frequency (transverse mode) or double the Strouhal frequency (longitudinal mode). The aerodynamic damping for transverse oscillation becomes negative in the synchronization range. In the case of tandem cylinders, at low Reynolds numbers in the pure Kármán range synchronization was observed to occur only when the downstream cylinder oscillated inside the vortex-formation region of the upstream one, and at high (low subcritical) Reynolds numbers synchronization occurred irrespective of the cylinder spacing in either oscillating mode. In the tandem case, too, the transverse oscillation of the downstream cylinder becomes unstable in the range of synchronization.

1. Introduction

A considerable amount of discussion is found in the literature concerning the wind-induced oscillations of smoke-stacks, cables and other structural forms. Information about the fluctuating forces on an oscillating cylinder is of special interest in aeroelasticity as well as for a basic understanding of fluid mechanics. However, only a few studies have been made for a circular cylinder oscillating in a uniform flow. Bishop & Hassan (1964), Jones (1968) and some others measured the lift and drag forces acting on a circular cylinder made to oscillate in a direction perpendicular to the stream, and recently Okajima, Takata & Asanuma (1971, 1972) studied experimentally and numerically the viscous flow around a torsionally oscillating circular cylinder. In these studies, the existence of aeroelastic resonance was confirmed.

There is a growing interest concerning another kind of instability that occurs in a wake from an upstream obstacle. For example, under certain flow conditions, serious aeroelastic instabilities can be observed in the array of pipes of a heat exchanger (Funakawa 1968) and in conductor bundles (Calvert 1969). These

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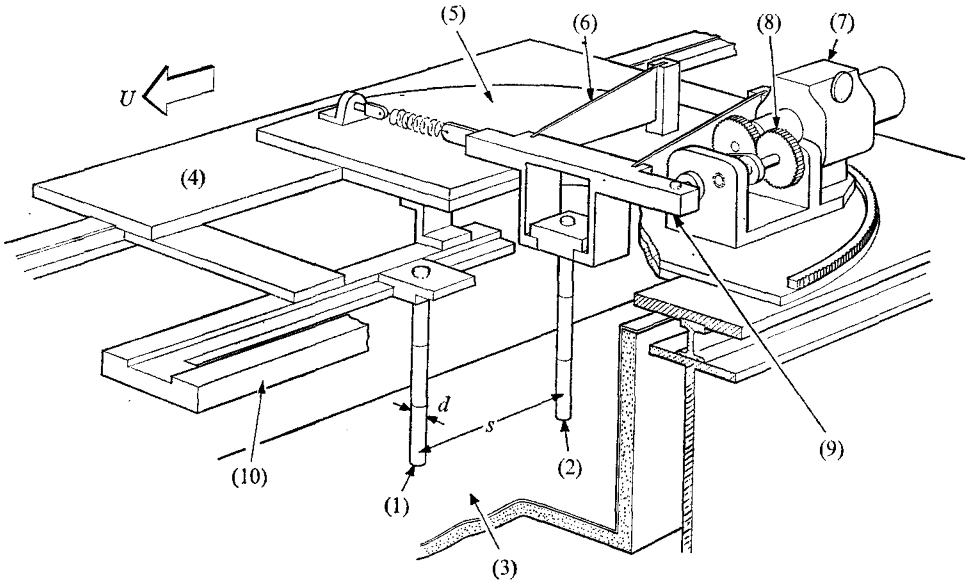

FIGURE 1. Experimental set-up.

instabilities may be attributable not only to the forced oscillation of the downstream cylinder due to the non-uniformity and unsteadiness of the wake flow, but also to the resonance in the aeroelastic system.

The aim of the present study is to measure the fluctuating lift and drag forces acting on a circular cylinder which oscillates either in uniform flow or in a wake, and to assess the possibility of aeroelastic instability, which may occur when the aerodynamic damping force becomes negative. The results obtained are for the following cases.

(*a*) A single circular cylinder either stationary or oscillated in one direction, perpendicular or parallel to the stream.

(*b*) A tandem arrangement with the upstream circular cylinder at rest and the downstream one either stationary or oscillated in a manner similar to that in the case (*a*).

The range of Reynolds numbers covered in this experiment is 40–10^4.

2. Experimental apparatus

2.1. *General remarks*

As shown in figure 1, the test cylinders (1) and (2), hanging vertically from the carriage (4), are towed in a still-liquid tank (3), which is 0·7 m wide, 0·4 m deep and about 10 m long. The carriage is pulled on tracks by towing ropes, which are wound round a drum by an electric variable-speed motor. Hence the uniformity of the velocity distribution in the spanwise direction of the cylinder is sufficiently good, and a low rate of turbulence in the flow can be expected.

The effects of the free liquid surface and the clearance between the cylinder and the tank bottom† scarcely affect the measurements as long slender cylinders are used. This was confirmed by a preliminary test, in which the effects of end plates were examined.‡ Furthermore, the measurements are performed on the central section of the test cylinder, so as to minimize three-dimensional flow effects.

In the tandem arrangement, one cylinder is held fixed upstream, and another is placed directly downstream of it. The downstream cylinder, (2) in figure 1, can be made to oscillate transversely with a simple harmonic motion by an oscillating mechanism on the carriage; that is, an electric variable-speed motor (7) drives the oscillation of the test cylinder through a gear train (8) and Scotch-yoke mechanism (9) suspended by parallel leaf springs (6). The amplitude of oscillation is selected as 4·2 mm, and the direction of oscillation, either perpendicular or parallel to the stream, can be changed by rotating the turntable (5). The upstream cylinder (1) is mounted at an arbitrary point on the bracket (10), and the distance between the centres of the two cylinders is adjustable in the range 2·5–20 cylinder diameters. In the case of a single cylinder, only the downstream cylinder is used, the upstream one being removed.

The use of water or oil as the fluid makes it possible to measure the unsteady aerodynamic forces on the oscillating cylinder with reasonable precision. The liquid temperature can be made as uniform as is necessary, by insulating the tank. The available Reynolds numbers are thus about 40–150 for oil and 10^3–10^4 for water.

The time for each experimental run is more than 20 s, being sufficiently long for the measurements to be performed with no detectable transient starting effects.

2.2. *Test cylinders*

Figure 2 shows a test cylinder, which is 30 mm in diameter and 352 mm long, being divided into three. The upper and lower sections (a) and (c) are fastened to each other by four parallel connecting rods (f). The central section (b) is 100 mm long and suspended from the upper dummy cylinder by two parallel leaf springs (d) of phosphor bronze on which two pairs of semi-conductor strain gauges (e) are patched to form a Wheatstone bridge that is sensitive to the force acting in one direction only. The leads to the strain gauges run out through the centre hole of the upper cylinder. The gauges and the leads are protected from the liquid by a soft wax coating.

The central section of the cylinder is made of hollow acrylic plastic, so that the unsteady aerodynamic forces acting on it can be deduced with a reasonable accuracy neglecting the inertial force even when the cylinder is oscillated. The natural frequency of the central section is around 40 Hz in water, which is

† The bottom of the tank is made flat but slightly inclined for drainage, so that the clearance between the cylinder and the bottom of the tank varies continuously between 3 and 20 mm.

‡ Two thin circular end plates 80 mm in diameter were attached to the cylinder, at the free end of the cylinder and at a position just under the free surface of the liquid. The effect of the end plates on the measurements could scarcely be detected, so the present experiments were carried out without the end plates.

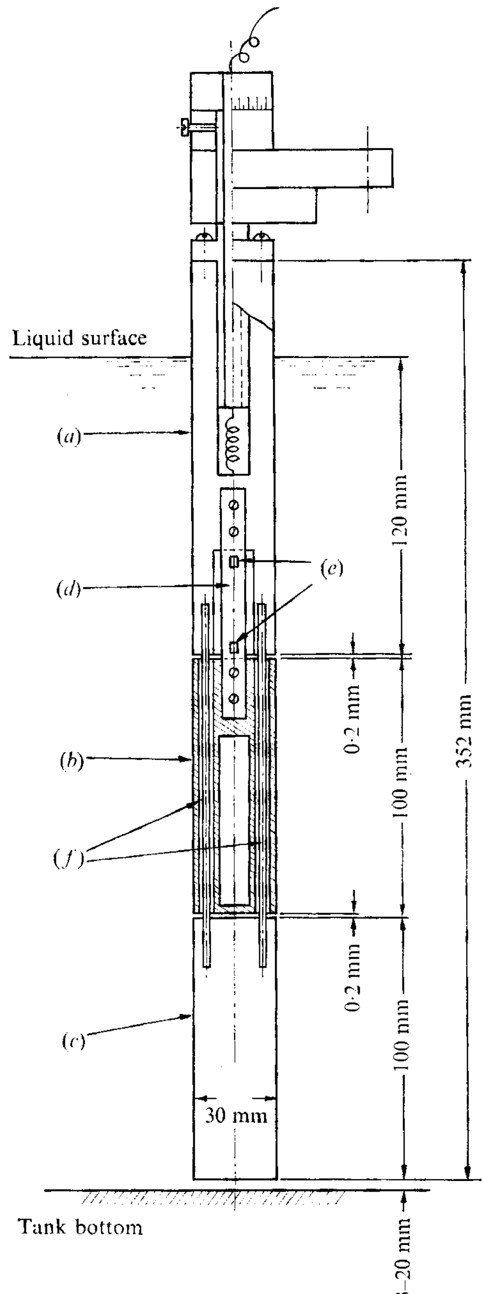

FIGURE 2. Details of test cylinder.

satisfactorily high in the present study. The gaps between these three sections are adjusted to be 0·2 mm or less; these were also checked by a preliminary test to have no perceptible influence on the measurements.

2.3. *Instrumentation*

In the present experiment, the lift and drag forces exerted on the cylinder are measured in separate runs. As mentioned previously, these forces are detected by the strain gauges fitted upon the beams supporting the central section of the test cylinder. The use of semi-conductor strain gauges of the *p–n* type makes the overall sensitivity of the system very high without loss of rigidity. The signals from the strain gauges are recorded by an ultraviolet recorder through amplifiers and low-pass filters.

When the direction of the cylinder oscillation coincides with that of the force to be measured, it often becomes difficult to deduce the fluctuating lift or drag force at high frequency. In this case, the inertial forces of the real and apparent mass are subtracted electrically from the overall signals, and only the part of the force in phase with the oscillating velocity of the cylinder is examined. This is similar to the method adopted by Bishop & Hassan (1964), and is sufficient for the assessment of the instability.

In operation, the driving frequency of oscillation is set in advance by reading directly the photoelectric tachometer, which counts the revolutions of the motor driving the oscillating mechanism. The displacement of the oscillating cylinder is also recorded by the ultraviolet recorder. Further, for reference, the frequency of vortex shedding is detected by a hot-wire anemometer whose probe is immersed in the wake of the cylinder.

2.4. *Method of data reduction*

The lift and drag forces per unit length for a cylinder are expressed respectively as

$$L = \tfrac{1}{2}\rho U^2 d\tilde{C}_L, \quad D = \tfrac{1}{2}\rho U^2 d(\bar{C}_D + \tilde{C}_D),$$

where ρ is the density of the fluid, U is the stream velocity, d is the diameter of the cylinder, C_L and C_D are the non-dimensionalized lift and drag forces, and the overbar and tilde denote steady and fluctuating components, respectively.

In uniform flow, a stationary circular cylinder is subjected to, besides the mean drag force, fluctuating lift and drag forces, with frequencies f_k and $2f_k$ respectively, owing to the alternating vortex shedding. As is well known, this can be characterized by a non-dimensional parameter $S_k = f_k d/U$ known as the Strouhal frequency.

When the cylinder is forced to oscillate with a frequency f_c that is appreciably smaller or larger than the Strouhal frequency, the fluctuating forces consist of two parts: one is due to the vortex shedding (whose frequency $f_v \doteqdot f_k$) and the other results from the cylinder oscillation. When the driving frequency of the cylinder approaches the Strouhal frequency, the two sets of forces become synchronized, and the cylinder and wake system oscillates at the imposed frequency f_c, when $f_v = f_c$. Then the fluctuating lift and drag forces oscillate regularly with fairly constant amplitudes. Outside the range of synchronization as well as in the stationary case, the amplitudes of the forces are not always constant, changing with nearly periodic beating wave forms. The records are then processed by averaging the amplitudes over a long period. The frequencies of the cylinder oscillation and the vortex shedding will be reduced into non-dimensional form as $S_c = f_c d/U$ and $S_v = f_v d/U$ respectively.

The stability of the cylinder oscillation will be examined from a purely aerodynamic point of view, and it can be assessed from the work done by the hydrodynamic forces in the direction of oscillation. For nearly sinusoidal force fluctuations, or assuming that the fluctuating force $\tilde{C} = [\tilde{C}] \cos (2\pi f_c t)$ for a cylinder oscillation $a \cos (2\pi f_c t)$, the stability of the cylinder oscillation depends on the force component in phase with the oscillating velocity, i.e. the instability is probable when $\mathrm{Im}\,[\tilde{C}] > 0$, which means negative aerodynamic damping.

In the following the results will be shown mostly for Reynolds numbers of 80 and 4000 (or 3400), being typical members of the ranges of low and high Reynolds numbers in the present study. The Reynolds number is of course defined as usual as $Re = Ud/\nu$, where ν is the kinematic viscosity.

3. Results for a single circular cylinder

3.1. *Preliminary remarks on a single stationary cylinder*

For reference in the evaluation of the results shown later, a preliminary experiment was performed on a single stationary circular cylinder in uniform flow.

As is expected from a number of previous works, the fluctuations in lift force are not observed for $Re < 40$, which means that there is no vortex shedding behind the cylinder. In the pure Kármán range of Reynolds numbers, a regularly oscillating lift force with fairly constant amplitude can be seen. As the Reynolds number increases, the lift force becomes irregular, fluctuating with a beating wave form.

Figure 3 gives the measured lift and drag forces and the Strouhal frequency versus the Reynolds number, compared with the results of the previous work (Relf & Simmons 1924; Roshko 1953; Delany & Sorensen 1953; Tritton 1959; Gerrard 1961). For the fluctuating lift, not only averaged values but also maxima and minima are shown in the figure. The fluctuating drag force is too small to measure accurately and is not shown here.

3.2. *Case of a cylinder oscillating transversely*

Figures 4 and 5 give the results for a single cylinder oscillating in the transverse (perpendicular to the stream) direction at $Re = 80$ and 4000, respectively, showing the reciprocal of the vortex-shedding frequency divided by the forcing frequency, S_c/S_v, the fluctuating lift component $\mathrm{Im}\,[\tilde{C}_L]$ in phase with the oscillating velocity, and the mean drag force $\bar{C}_D$. Similar responses are obtained in the ranges of low and high Reynolds number although there is a slight shift of the synchronization range between them.

When the oscillating frequency is either small or large enough, or with the frequency detuned more than 35 % away from the Strouhal frequency, the vortex shedding occurs at the natural Strouhal frequency irrespective of the

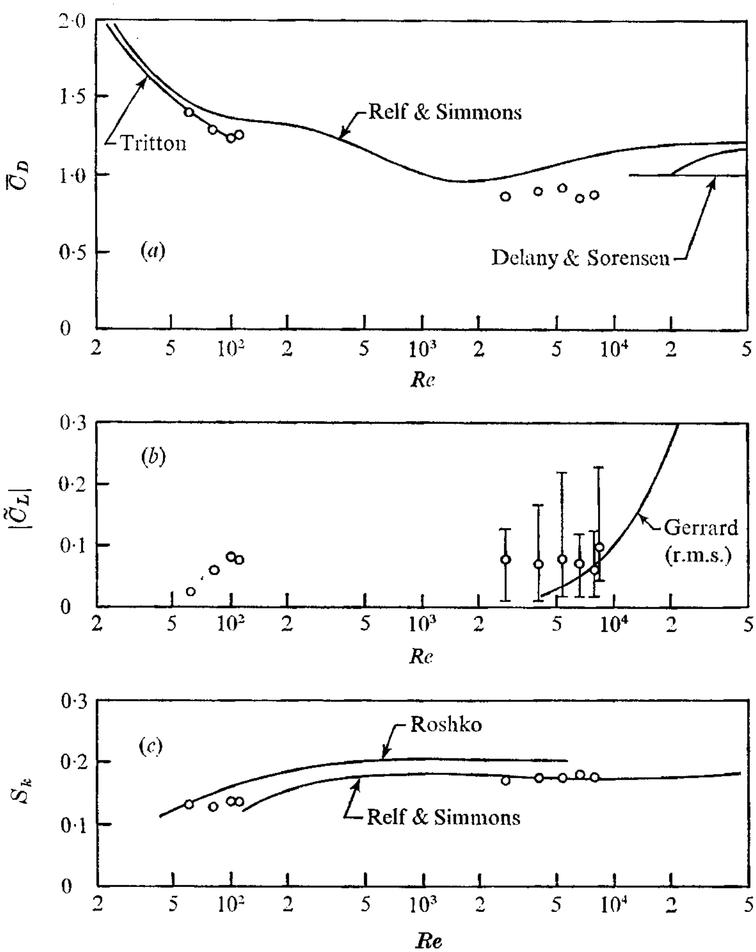

FIGURE 3. Characteristics of a single stationary cylinder. (*a*) Mean drag; (*b*) lift; (*c*) Strouhal frequency.

oscillation, and the two sets of fluctuating forces are superposed upon each other. As the driving frequency approaches the Strouhal frequency, however, the system is synchronized to oscillate at the forcing frequency, and vortices are shed with frequency f_c or $S_c/S_v = 1$. The synchronization occurs over a finite frequency range, in which (*a*) the lift force oscillates regularly with fairly constant amplitude and takes a maximum value at around the centre of the synchronization range, being accompanied by a sudden phase lag, and (*b*) the mean drag force also takes a maximum value at around the centre of the range.

Hence, in the lower half of the synchronization range, the fluctuating lift component $\mathrm{Im}\,[\tilde{C}_L]$ in phase with the oscillating velocity becomes positive, as shown in figures 4(*b*) and 5(*b*), and then it is possible for the single cylinder to oscillate autonomously owing to the hydrodynamic excitation. These results agree with those of the previous work, e.g. by Tanaka & Takahara (1970).

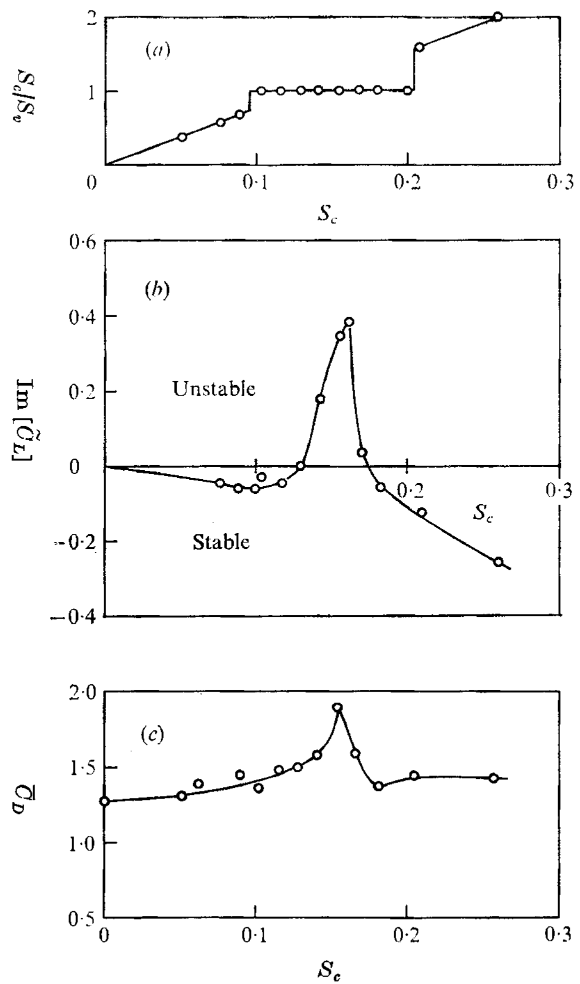

FIGURE 4. Case of a single cylinder oscillating transversely. $Re = 80$. (*a*) Oscillation/vortex-shedding frequency ratio; (*b*) in-phase lift component; (*c*) mean drag.

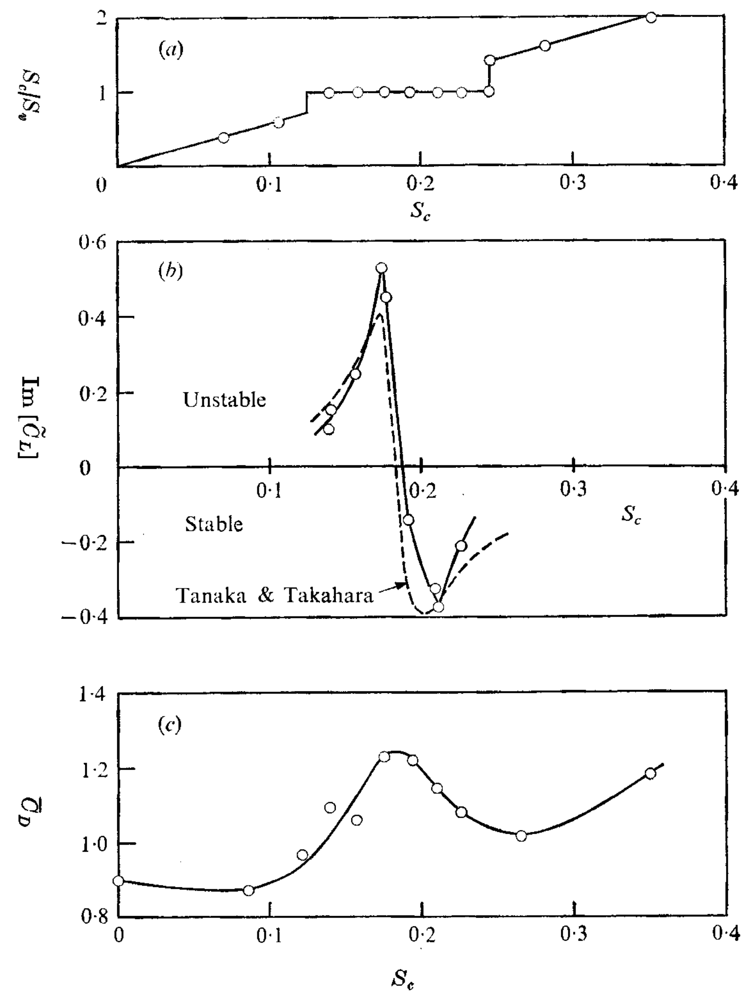

FIGURE 5. Case of a single cylinder oscillating transversely. $Re = 4000$. (*a*) Oscillation/vortex-shedding frequency ratio; (*b*) in-phase lift component; (*c*) mean drag.

3.3. *Case of a cylinder oscillating longitudinally*

In figures 6 and 7 the results for a circular cylinder oscillating in the longitudinal (parallel to the stream) direction in uniform flow are given. Synchronization can be observed in a range around double the Strouhal frequency, where vortices are shed with a frequency half the imposed one. Synchronization of this type has been already found by Tatsuno (1972) in his flow visualization experiment.

Figures 6 and 7 show that the fluctuating lift and mean drag forces take maxima in the middle of the synchronization range, as in the case of transverse oscillation. In figure 7 (*b*) for the higher Reynolds number, the vertical lines in the synchronization range denote that periods in which the lift fluctuation vanishes arise alternatively. Then it is probable that the oscillation of this mode acts not only to excite the vortex shedding but also to suppress it occasionally. The fluctuating drag component $\mathrm{Im}\,[\tilde{C}_D]$ in phase with the oscillating velocity is always negative in this case, so that the longitudinal oscillation is stable from the autonomous point of view as far as the present experiment is concerned. However, the instability is likely to occur at much higher Reynolds number, because $\mathrm{Im}\,[\tilde{C}_D]$ tends to become positive in the synchronization range as the Reynolds number increases, deviating from the quasi-steady value as shown in figure 7 (*c*).

4. Results for tandem circular cylinders

4.1. *Case of both cylinders at rest*

Figures 8 and 9 give the measured mean drag and fluctuating lift forces acting on two stationary cylinders in tandem at $Re = 80$ and 3400 respectively, together with the Strouhal frequency, for separations up to 20 diameters, i.e. for $s \leqq 20d$. At the higher Reynolds number, the fluctuating lift on both cylinders changes with a beating wave form, the maxima and minima of which are shown by vertical lines in figure 9 (*b*).

For separations smaller than 5 diameters at $Re = 80$ and 3 diameters at $Re = 3400$, no fluctuations in lift can be observed. This implies that the downstream cylinder suppresses the vortex shedding from the upstream one, playing a role like that of a splitter plate behind a cylinder. This is supported by Thomas & Kraus's (1964) results of flow visualization for $s/d = 3{\cdot}6$ at $Re = 62$. It is noted that the critical separation for the vortex shedding, $s/d \doteqdot 5$ at $Re = 80$ and $s/d \doteqdot 3$ at $Re = 3400$ in the present case, is consistent with the length of the so-called vortex-formation region given in the previous work (Schaefer & Eskinazi 1959; Bloor & Gerrard 1966).

For separations larger than the critical value, vortices are shed periodically from the upstream cylinder, and both cylinders undergo fluctuations in lift and drag forces with common frequencies. As the separation increases, the amplitude $|\tilde{C}_{L1}|$ of the fluctuating lift acting on the upstream cylinder soon settles at the value for a single stationary cylinder (indicated by an arrow in the figures). On the downstream cylinder, however, the lift amplitude $|\tilde{C}_{L2}|$ sharply increases with increasing separation and reaches a maximum that is 8 to 10 times as large

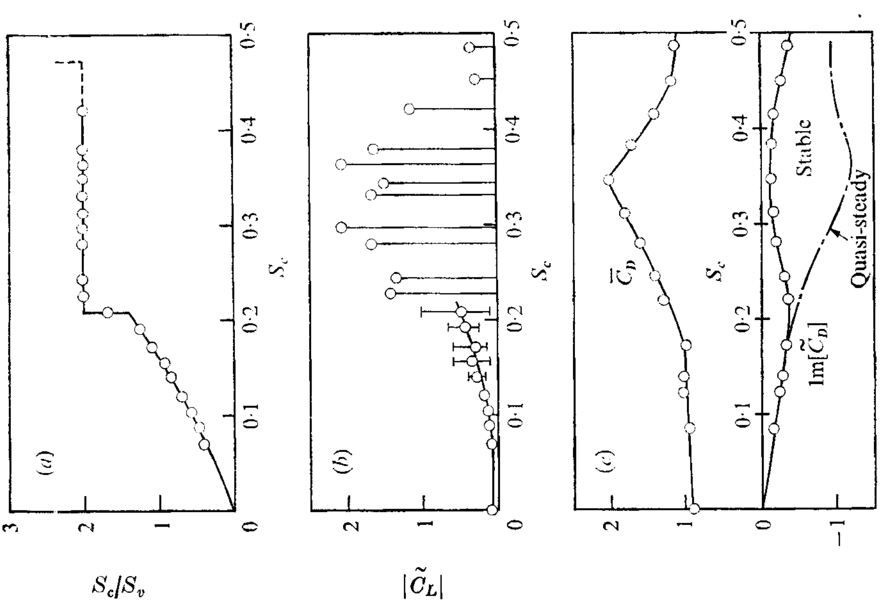

FIGURE 7. Case of a single cylinder oscillating longitudinally. *Re* = 4000. (*a*) Oscillation/vortex-shedding frequency ratio; (*b*) lift; (*c*) mean drag and in-phase drag component.

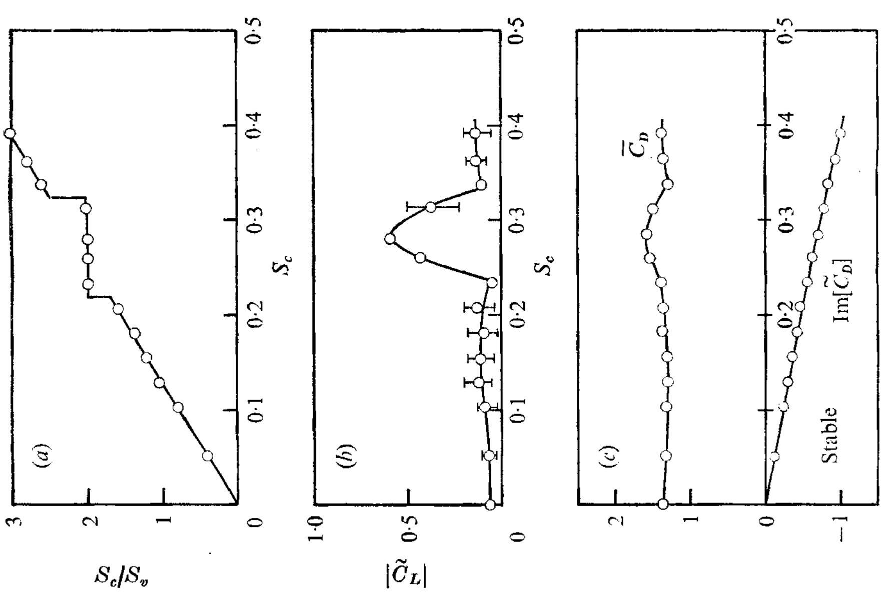

FIGURE 6. Case of a single cylinder oscillating longitudinally. *Re* = 80. (*a*) Oscillation/vortex-shedding frequency ratio; (*b*) lift; (*c*) mean drag and in-phase drag component.

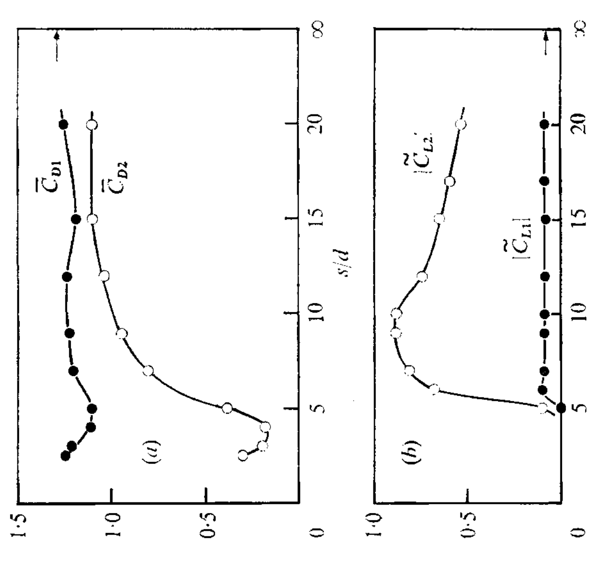

FIGURE 8. Characteristics of tandem stationary cylinders. $Re = 80$. (*a*) Mean drag; (*b*) lift; (*c*) Strouhal frequency.

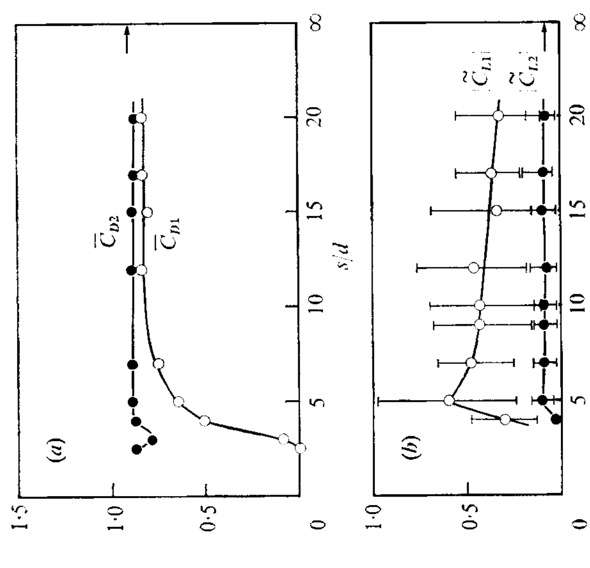

FIGURE 9. Characteristics of tandem stationary cylinders. $Re = 3400$. (*a*) Mean drag; (*b*) lift; (*c*) Strouhal frequency.

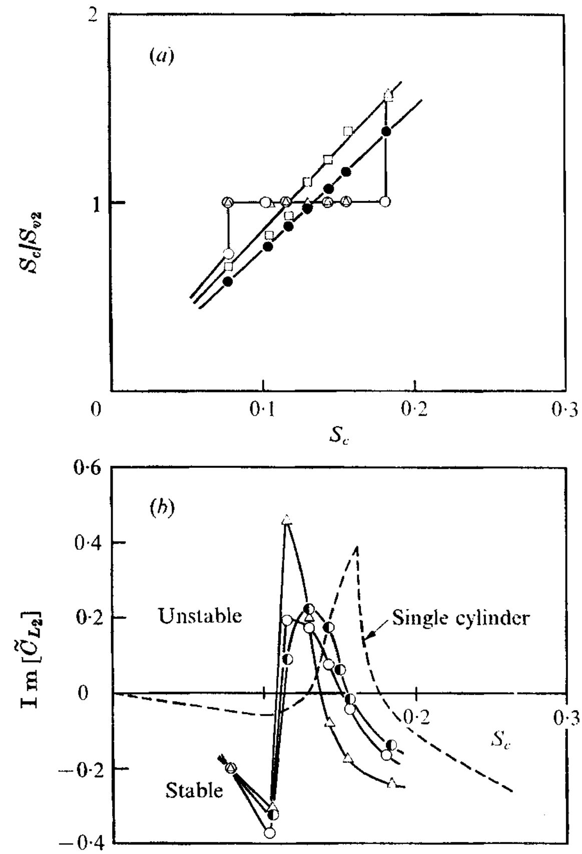

FIGURE 10. Case of downstream cylinder oscillating transversely. $Re = 80$. (a) Oscillation/vortex-shedding frequency ratio; (b) in-phase lift components on the downstream cylinder. ◐, $s/d = 2{\cdot}5$; ○, $s/d = 3$; △, $s/d = 4$; □, $s/d = 5$; ●, $s/d = 15$.

as that for a single cylinder, and remains large even at $s/d = 20$, although it gradually decreases until it reaches the value for a single cylinder at infinite separation.

The Strouhal frequency S_k in this case is slightly lower for small separations than in the case of single cylinder, but it soon recovers with increasing separation. At $Re = 3400$, it is seen that S_k increases again for very small separations near the critical point.

As shown in figures 8 (a) and 9 (a), in both the range of low and of high Reynolds numbers, the mean drag force $\bar{C}_{D1}$ acting on the upstream cylinder is slightly subject to the influence of the downstream one only when the separation between them is very small. On the other hand, the front surface of the downstream cylinder is strongly affected by the negative pressure of the upstream wake, so that its mean drag force $\bar{C}_{D2}$ decreases remarkably at small spacings. At much higher Reynolds numbers, the remarkable decrease in the mean drag may occur on the downstream cylinder over a wide range of separations (Biermann & Herrnstein 1933).

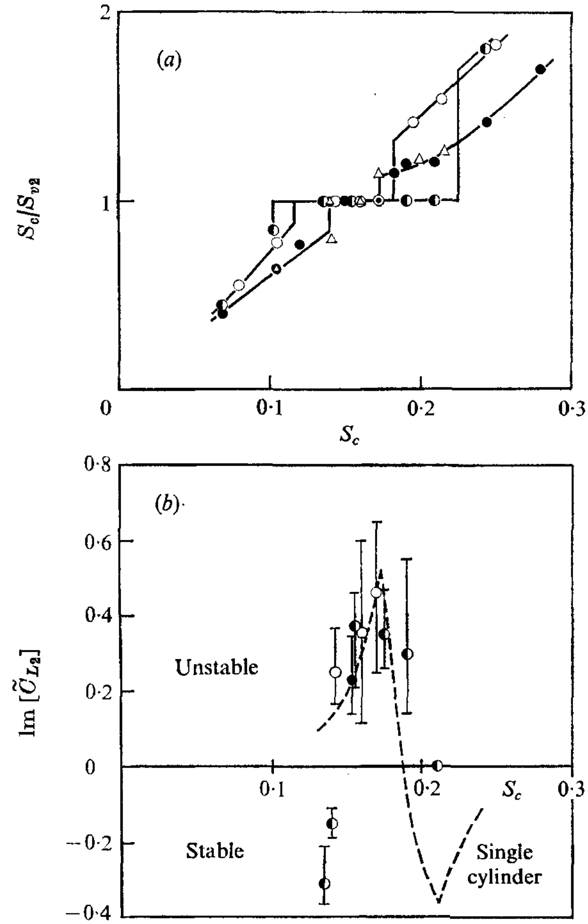

FIGURE 11. Case of downstream cylinder oscillating transversely. Re = 4000. (a) Oscillation/vortex-shedding frequency ratios; (b) in-phase lift components of the downstream cylinder. ◐, $s/d = 2{\cdot}5$; ○, $s/d = 3$; △, $s/d = 4$; ●, $s/d = 10$.

4.2. *Case of downstream cylinder oscillating transversely*

Figure 10 shows that, when the downstream cylinder oscillates transversely at $Re = 80$, synchronization does not occur for separations $s/d > 5$; that is, irrespective of the oscillation of the downstream cylinder, vortices are shed from the upstream cylinder with the natural Strouhal frequency as in the case of both cylinders at rest, being washed downstream without being much influenced by the downstream cylinder. For separations $s/d < 5$, when no vortex shedding is observed in the stationary case, the oscillation of the downstream cylinder forces vortex shedding, just as an aerofoil with an oscillating trailing edge does, and brings about synchronization. In the range of synchronization, the amplitudes of the fluctuating lifts on the two cylinders increase simultaneously, and the in-phase lift component $\mathrm{Im}\,[\tilde{C}_{L2}]$ on the downstream oscillating cylinder becomes positive, which leads to the possibility of instability as in the case of a single cylinder.

Figure 11 shows that at $Re = 4000$ synchronization can be detected for all

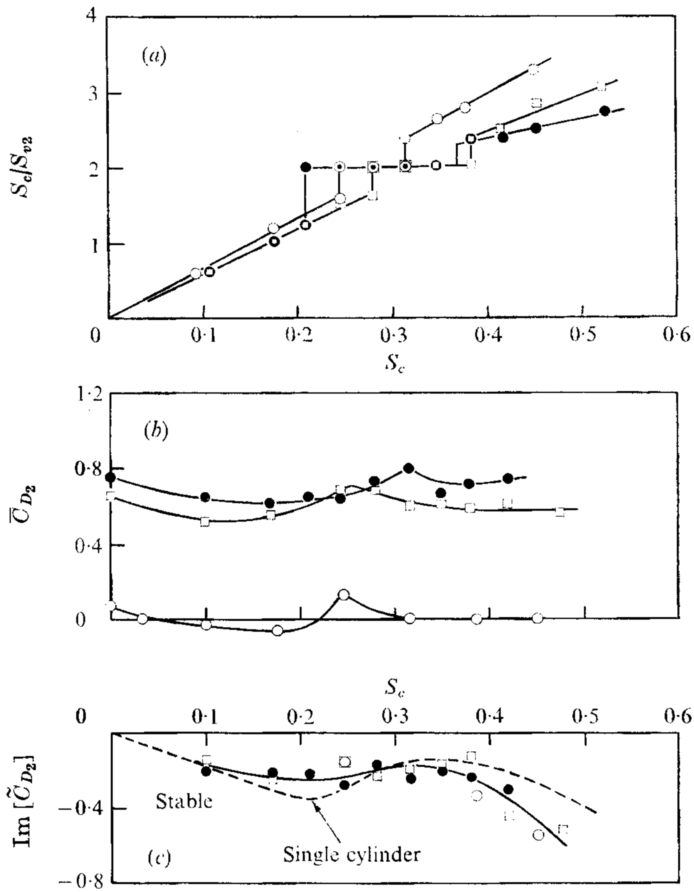

FIGURE 12. Case of downstream cylinder oscillating longitudinally. $Re = 4000$. (*a*) Oscillation/vortex-shedding frequency ratios; (*b*) mean drag on the downstream cylinder; (*c*) in-phase drag components on the downstream cylinder. ○, $s/d = 3$; □, $s/d = 5$; ●, $s/d = 7$.

cylinder separations with which the present experiments is concerned, although it is more or less random and accompanied by temporary pauses. When the downstream cylinder is located inside the vortex-formation region of the upstream one, or for $s/d < 3$, the fluctuating lifts on both cylinders synchronize with the imposed oscillation. However, when the downstream cylinder is located far downstream, or for $s/d > 3$, the fluctuating lift on the downstream cylinder synchronizes with the oscillation, whereas the upstream cylinder is scarcely affected by the oscillation of the downstream one. Figure 11(*b*) gives the in-phase lift component Im $[\tilde{C}_{L2}]$ acting on the downstream cylinder, showing the negative aerodynamic damping effect in the range of synchronization as in the case of the single cylinder.

4.3. *Case of downstream cylinder oscillating longitudinally*

At low Reynolds numbers such as $Re = 80$, synchronization cannot be induced by longitudinal oscillation of the downstream cylinder for any separation in the experiment. For smaller separations such as $s/d < 5$, no vortex shedding is observed in spite of the oscillation of the downstream cylinder, and for $s/d > 5$, vortices are shed almost regardless of the oscillation, and the fluctuating lifts on both cylinders change at a common frequency. There is no remarkable feature in the fluctuating drag component in phase with the oscillating velocity, and it is almost proportional to the oscillating frequency, yielding a positive damping effect.

At a high Reynolds number $Re = 4000$, figure 12 shows that synchronization can be detected for any separation over 3 diameters, when the downstream cylinder may be outside the vortex-formation region of the upstream one. However, the effect of synchronization is so limited that vortices are shed from the upstream cylinder as in the case of single cylinder, while the vortex shedding from the downstream cylinder is performed at half the driving frequency. Hence, the synchronization does not greatly affect the mean drag forces on both cylinders, and the damping force $\mathrm{Im}\,[\tilde{C}_{D2}]$ acting on the downstream cylinder is always negative, although it has a tendency to become positive owing to synchronization.

5. Concluding remarks

The stability of a circular cylinder oscillating in uniform flow or in a wake has been studied by measuring the lift and drag forces in the case of a single cylinder and tandem cylinders. It was found that, in either case, the aerodynamic damping force becomes negative when a cylinder (the downstream one in the tandem case) oscillates transversely in the synchronization range; that is, the transverse oscillation of the cylinder may become unstable when the cylinder motion and the vortex shedding are synchronized.

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